Linear Inequalities
Worksheet to Accompany Videotape #9

Rules

1. We can add or subtract the same number on both sides of an inequality and have the same inequality sign.

2. We can multiply or divide both sides of an inequality by the same positive number and have the same sign.

3. Multiplying (or dividing) both sides of an inequality by a NEGATIVE NUMBER REVERSES THE INEQUALITY SIGN.

Examples

2 < 5 also
2+3 < 5+3 also
2-4 < 5-4

-1 < 3 also
-1(2) < 3(2) also
-1(1/4) < 3(1/4)

3 < 7 but
3(-1) > 7(-1)

To solve a linear inequality means to find all values of x which satisfy the inequality. Method is to use above rules to get x alone on one side.

Example:

5x - 3 < 8x - 12
+3 +3
5x < 8x - 9
-8x -8x
-3x < -9

Now divide both sides by -3 and Reverse Sign

-3x > -9
-3

x > 3

Set notation: \{x \mid x > 3\}

Graph:

Interval notation: (3, + ∞)

I. For you to try:

1. 3x - 27 > 6
2. 2x + 5 > 4x - 9
3. \frac{x}{3} - 2 < \frac{5x + 9}{2}
4. 2x + 5 < 3
5. 6 - 4y > 4 - 3y
6. -6(2+r) < 2(3r - 6)
7. 3 - 6c > 15
8. 3(3-k) - 10 > 2
9. 4x - 5 > 8x + 1
10. 3(x - 4) > \frac{x}{2} - 6
Example: Solve \(-3 < 2x + 5 < 7\). We use rules to get an expression of the form \(a < x < b\).

Steps of solution:

\[
\begin{align*}
-3 &< 2x + 5 < 7 \\
-5 &< 2x < 2 \\
-4 &< x < 1
\end{align*}
\]

Set notation: \(\{x \mid -4 < x < 1\}\)

Graph:

Interval Notation: \((-4, 1)\)

II. For you to try:

1. \(4 \leq 2x - 5 \leq 10\)
2. \(-4 < 3x - 2 \leq 10\)
3. \(-4 < 2x + 9 < 16\)
4. \(6 \leq \frac{1}{2}(x + 3) < 10\)
5. \(-3 \leq 5x + 1 \leq 3\)
6. \(-4 \leq 6 - 2x < 4\)
Answers: For you to try:  (Graphs not drawn to scale.)

I. 1. \( \{ x \mid x > 11 \} \), \( (11, + \infty) \)  
2. \( \{ x \mid x < 7 \} \), \( (-\infty, 7) \)  
3. \( \{ x \mid x > -3 \} \), \( (-3, +\infty) \)  
4. \( \{ x \mid x < -1 \} \), \( (-\infty, -1) \)  
5. \( \{ y \mid y < 2 \} \), \( (-\infty, 2) \)  
6. \( \{ r \mid r > 0 \} \), \( (0, +\infty) \)  
7. \( \{ c \mid c < -2 \} \), \( (-\infty, -2) \)  
8. \( \{ k \mid k < -1 \} \), \( (-\infty, -1) \)  
9. \( \{ x \mid x < -\frac{3}{2} \} \), \( (-\infty, -\frac{3}{2}) \)  
10. \( \{ x \mid x > \frac{12}{5} \} \), \( (\frac{12}{5}, +\infty) \)

II. 1. \( \{ x \mid \frac{9}{2} \leq x \leq \frac{15}{2} \} \), \( \left[ \frac{9}{2}, \frac{15}{2} \right] \)  
2. \( \{ x \mid -\frac{2}{3} < x \leq 4 \} \), \( (-\frac{2}{3}, 4] \)  
3. \( \{ x \mid -\frac{13}{2} < x < \frac{7}{2} \} \), \( (-\frac{13}{2}, \frac{7}{2}) \)  
4. \( \{ x \mid 9 \leq x < 17 \} \), \( (9, 17) \)  
5. \( \{ x \mid -\frac{4}{5} \leq x \leq \frac{2}{5} \} \), \( \left[ -\frac{4}{5}, \frac{2}{5} \right] \)  
6. \( \{ x \mid 1 < x \leq 5 \} \), \( (1, 5] \)