Composite Functions
Worksheet to Accompany Videotape #16

Let \( f \) and \( g \) be functions. The composite function, or composition, of \( g \) and \( f \) is

\[(gof)(x) = g[f(x)]\]

for all \( x \) in the domain of \( f \) such that \( f(x) \) is in the domain of \( g \).

Example: Let \( f(x) = x^2 + 2x \) and \( g(x) = -3x \). Find \( (gof)(3) \).

Find \( (gof)(x) \) and \( (fog)(x) \):

\[(gof)(x) = g[f(x)] = g[x^2 + 2x] = -3(x^2 + 2x) = -3x^2 - 6x\]

\[(fog)(x) = f[g(x)] = f(-3x) = (-3x)^2 + 2(-3x) = 9x^2 - 6x\]

1. Given \( f(x) = 4x^2 - 2x \) and \( g(x) = 8x + 1 \), find:
   a. \( (gof)(2) \)
   b. \( (fog)(-1) \)

2. Find \( fog \) and \( gof \) for each of the following pairs of functions:
   a. \( f(x) = 8x + 12 \), \( g(x) = 3x - 1 \)
   b. \( f(x) = 5x + 3 \), \( g(x) = -x^2 + 4x + 3 \)
   c. \( f(x) = -x^3 + 2 \), \( g(x) = 4x \)
   d. \( f(x) = \sqrt{x + 2} \), \( g(x) = 8x^2 - 6 \)
   e. \( f(x) = \frac{1}{x - 5} \), \( g(x) = \frac{2}{x} \)

\[\text{ANSWERS: (1) a. 97 b. 210 (2) a. 2x + 4 b. 2x + 35 c. -6x + 2, -4x + 8 d. 8x + 10 \to 0, 8x - 4, 8 + 8, -2x + 18, -2x + 35, 2x + 4} \]